Diagonalization Principle

Let S be a non-empty set and R any relation on S.

Let

$D = \{a \epsilon A \mid (a,a) \ \epsilon \ r\}$

For each $a \epsilon A$, let R_a ={ b |(a,b) ϵR}

Then diagonalization principle states that D is different from each R_a.

OR

Diagonalization principle states that the complement of the diagonal is different

from each row

For example,

Let S = {a, b, c, d}
R = { (a,a), (b,c), (b,d), (c,a), (c,c), (c,d), (d,a), (d,b) }

The above relation R is shown in matrix from as follows:

	a	b	c	d
a	X			
b			X	X
c	X		X	X
d	X	X		

Diagonal elements are marked.

From the figure,

R_a = { a }

R_b = {c,d}

R_c = {a, c, d}

R_d = {a, b}

Diagonal D = {a,b}

Complement of Diagonal, D' = {b,d}

	a	b	c	d
a				
b		X		
c				
d				X

If we compare each of the above R_a, R_b, R_c and R_d with D, we can see that D is different from each.

Thus complement of the diagonal is distinct from each row.

Note:

Following information is needed to solve the example given below:

9's complement of a number 9's complement of 276 is 723.

9's complement of 425 is 574.

9's complement of 793 is 206.

Example 1:

Prove that the set of real numbers between 0 and 1 is uncountable.

An example for a real number between 0 and 1 is 0**.**34276

Let us represent a real number between 0 and 1 as

$X = 0.X_0X_1X_2X_3\ldots..$

where each x_i is a decimal digit.

Let f(k) be an arbitrary function from natural numbers to the set [0,1].
We can arrange the elements in a 2d array as,

f(0):	:	X_{00}	X_{01}	X_{02}	X_{03}	_	_	_
f(1):	:	X_{10}	X_{11}	X_{12}	X_{13}	_	_	_
f(2):	:	X_{20}	X_{21}	X_{22}	X_{23}	_	_	_
- - -		- - - - -						
- - - -		- - - - -						
f(n):	:	X_{n0}	X_{n1}	X_{n2}	X_{n3}	_	_	_

where x_{ni} is the i^{th} digit in the decimal expansion of f(n).

Next we find the complement of the diagonal (9's complement) as follows:
$Y = .y_0 y_1 y_2$ ....
where y_i = 9's complement of x_{ii}

(Find out the 9's complement of x_{00}, x_{11}, x_{22}, x_{33}…x_{nn}
From the diagonalisation principle, it is clear that the complement of the diagonal is different from each row.

Here it is clear that Y is different from each f(i) in at least one digit. $Y \neq f(i)$.

Hence Y cannot be present in the above array.

This means that the set of real numbers between 0 and 1 is countably infinite or not countable.

For instance suppose we arrange the real numbers as,

f(0):	.	9	4	2	4 - - -
f(1):	.	6	3	6	2 - - -
f(2):	.	2	8	6	4 - - -
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f(3)	.	6	5	3	2 - - -
f(n):	.	4	1	5	7 - - -

Here the diagonal is 9 3 6 2

The complement (9’s complement) of the diagonal is 0 6 3 7 ....

The real number .0637... is not in the above table.

Thus It is clear that this value is distinct from each row, f(0), f(1), f(2),...f(n).